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OPTIMISING THE THERMOHYDRAULIC PERFORMANCE OF ROUGH SURFACES

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Abstract—The parameters which control the momentum- and heat-transfer performance of a rough surface in a uniform channel flow are presented in a novel way. A new efficiency parameter is defined for optimising this performance. Using a recently developed analysis a wide range of rough surfaces are investigated and design charts presented.

NOMENCLATURE

b, effective width of roughness element;
B^{-1}, Stanton number roughness parameter defined by equation (9);
c, characteristic separation length, $= c_3 + c_4$;
C^{-1}, efficiency roughness parameter defined by equation (13);
C_D, form drag coefficient;
D, hydraulic diameter;
g, heat-transfer roughness function;
h, height of roughness element;
h^+, roughness Reynolds number;
l^+, laminar sub-layer thickness, $\approx 11{\cdot}0$;
L^{-1}, new efficiency parameter defined by equation (16);
Pr, Prandtl number;
Pr_t, turbulent Prandtl number, $\approx 0{\cdot}9$;
R, momentum-transfer roughness function;
$Re(D)$, channel Reynolds number;
St, Stanton number.

Greek symbols
λ, friction factor;
ε, efficiency parameter defined by equation (10).

Subscript
s, refers to hydraulically smooth surface.

1. INTRODUCTION

THE PURPOSE of this paper is to present a method and design charts for evaluating and optimising the thermohydraulic performance of rough surfaces in channels where the concept of an equivalent hydraulic diameter is valid. Rough surfaces play an important role in many engineering heat-transfer problems [1] because they can make a particular heat-transfer device more efficient [2]. The fundamental parameters which control the momentum- and heat-transfer performance of a rough surface are the roughness functions [3] R and g, respectively. Until recently these were determined solely by experiment and, because of the large number of independent variables which govern the performance problem, this has led to an enormous quantity of published, often inconsistent [4], empirical information. This makes it almost impossible to optimize or generalize this information.

We attempt a more unified approach by making use of an analysis [5] which is based upon an approximate model of the separated flow over each roughness element. We use this to investigate the performance of a wide range of rough surfaces in a range of channel flows. The engineer or designer can use the results as a first approximation to find the surface which appears most suitable for his purposes. Then only a small number of fundamental experiments need be carried out to confirm or modify the predicted performance.

2. BASIC ASSUMPTIONS AND LIMITATIONS OF THE ANALYSIS

The analysis [5] applies strictly to the following situation: (i) the steady, fully-developed, turbulent flow of an incompressible, constant property, single-phase, Newtonian fluid; (ii) axial flow in a channel of constant cross-section with uniformly roughened walls and with an equivalent hydraulic diameter D; (iii) a characteristic roughness height h very much less than D; (iv) a constant, uniform wall heat flux; (v) a Prandtl number of order unity or greater; and (vi) two-dimensional transverse roughness. The theoretical model was originally developed for a roughness having the form of equally-spaced, rectangular ribs, Fig. 1. The ribs were

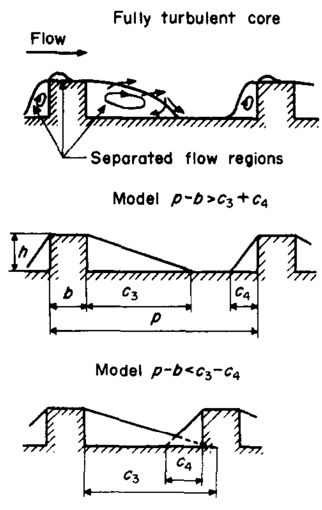

FIG. 1. The physical flow over roughness elements and the approximate models.

characterized by their geometry—pitch p, width b and height h—and a form drag coefficient C_D. Separated flow regions at the front and rear of each element were characterized by a separation length $c = c_3 + c_4$. The model was extended to other shapes, including three dimensional elements, by means of form drag coefficients [6] for these elements and by invoking the concept of "equivalent-rectangular-rib-geometry", Fig. 2. This is estimated from the separated flow regions for the non-rectangular shapes. The concept becomes progressively more invalid for shapes which differ markedly from rectangular ribs, such as elements with very low values of C_D. With these elements the separated flow regions are almost certainly no longer similar to those formed on rectangular ribs.

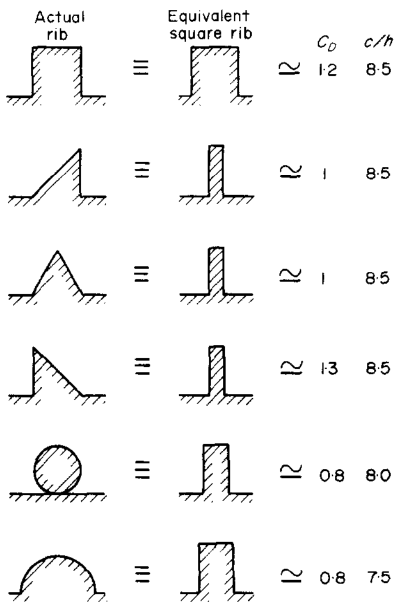

FIG. 2. Equivalent geometries and drag coefficients, after Hoerner [6].

Here, rather than taking specific shapes in the parameter study, we choose a range of representative form drag coefficients and a separation length together with a wide range of values for p, h and b. Besides the uncertainty in the theoretical prediction [5] this is the main weakness of the present approach; even though we can mathematically specify a value for C_D and c/h it may be impossible to achieve these in practice. To reduce the number of independent variables involved a Prandtl number $Pr = 0{\cdot}66$ is assumed throughout. Other values of Pr could have been investigated.

3. THE INTEGRAL-PERFORMANCE EQUATIONS AND THEIR DEPENDENCE ON THE ROUGHNESS PARAMETERS

The engineer or designer wishes to know by how much the integral parameters—friction factor λ, Stanton number St and Reynolds number $Re(D)$—are affected by the surface roughness. We use the Prandtl two-layer integral model [7] for a turbulent pipe flow and the modified Reynolds analogy to obtain representative equations.

For a smooth wall,

$$(8/\lambda_s)^{1/2} = 2{\cdot}5\{\ln Re(D)(\lambda_s/8)^{1/2}/2\} + 1{\cdot}75 \qquad (1)$$

and

$$\frac{1}{St_s} = (8/\lambda_s)^{1/2}\{Pr_t(8/\lambda_s)^{1/2} + l_s^+(Pr^{2/3} - Pr_t)\} \qquad (2)$$

where Pr_t (assumed $= 0{\cdot}9$) is the turbulent Prandtl number and l_s^+ (assumed $= 11{\cdot}63$) is the laminar sublayer thickness and subscript s refers to smooth wall parameters.

For a rough wall,

$$(8/\lambda)^{1/2} = 2{\cdot}5 \ln D/2h + R - 3{\cdot}75 \qquad (3)$$

$$= 2{\cdot}5 \ln\{Re(D)(\lambda/8)^{1/2}/2h^+\} + R - 3{\cdot}75 \qquad (4)$$

where h^+ is the roughness Reynolds number and

$$Re(D) \equiv (8/\lambda)^{1/2} h^+ D/h. \qquad (5)$$

Finally,

$$\frac{1}{St} = (8/\lambda)^{1/2}\{Pr_t(8/\lambda)^{1/2} + g - Pr_t R\}. \qquad (6)$$

Once R and g are determined as a function of h^+ the equations may be solved for the integral parameters. The roughness functions are given by

$$R = R(h^+, p/h, h/b, c/h, C_D) \qquad (7)$$

and

$$g = g(h^+, p/h, h/b, c/h, C_D, Pr) \qquad (8)$$

these may be evaluated theoretically [5] or experimentally [3]. A major difficulty is the large number of variables involved. To present these in a clear manner and to bring out the important parameters the following novel approach is adopted.

The parameter common to equations (1)–(6) is λ and we choose this as our main independent variable. The equations are plotted in Fig. 3 with D/h, h^+D/h and the Owen and Thomson [8] function B^{-1} as secondary independent variables. B^{-1} is defined as

$$B^{-1} \equiv g - Pr_t R \qquad (9)$$

and R, St and $Re(D)$ are treated as dependent variables. The smooth wall equations and the laminar Hagen–Poiseuille equation are also shown on the graphs.

4. THE EFFICIENCY OF A ROUGH SURFACE

An important efficiency parameter [2] for rough surfaces is ε defined by

$$\varepsilon \equiv [St/St_s]^3/[\lambda/\lambda_s] \qquad (10)$$

where St_s and λ_s are determined at the same channel Reynolds number as St and λ. It is often desirable to optimize ε but, to do this, we need to know the

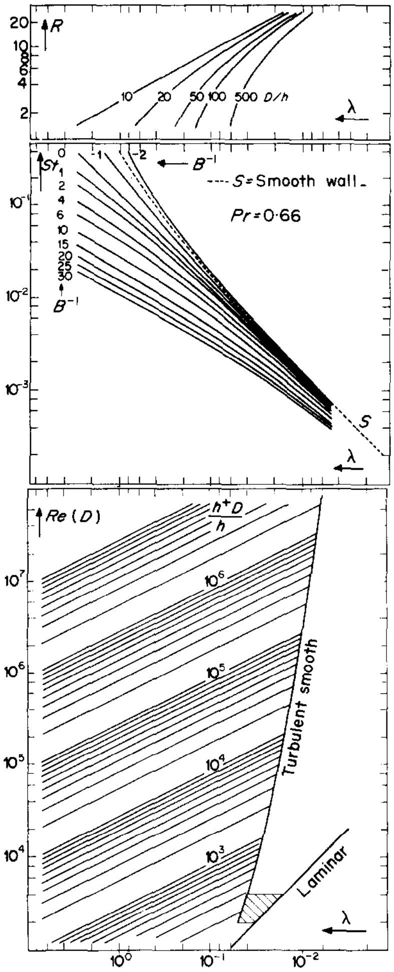

FIG. 3. Design chart for integral parameters λ, St, $Re(D)$ in terms of roughness parameters and D/h.

independent variables for ε. Eliminating $Re(D)$ from equations (1) and (4) gives

$$(8/\lambda)^{1/2}-(8/\lambda_s)^{1/2} = 2{\cdot}5\ln(\lambda/\lambda_s)^{1/2} - (2{\cdot}5\ln h^+ + 5{\cdot}5 - R) \quad (11)$$

$$= 2{\cdot}5\ln(\lambda/\lambda_s)^{1/2} - C^{-1} \quad (12)$$

where C^{-1} is defined by

$$C^{-1} \equiv 2{\cdot}5\ln h^+ + 5{\cdot}5 - R. \quad (13)$$

Hence,

$$\varepsilon = \varepsilon(\lambda, B^{-1}, C^{-1}) \quad (14)$$

and to optimize ε we must solve equations (2), (6) and (12) to see how ε varies with B^{-1} and C^{-1}. This was done for a wide range of representative values, and it was found, Fig. 4, that to a good approximation

$$\varepsilon = \varepsilon(\lambda, L^{-1}) \quad (15)$$

where L^{-1}, a unique optimization parameter, is defined by

$$L^{-1} \equiv C^{-1} - B^{-1} \equiv 2{\cdot}5\ln h^+ + 5{\cdot}5 - g - R(1 - Pr_t). \quad (16)$$

Very simply then, to maximize ε we need a maximum

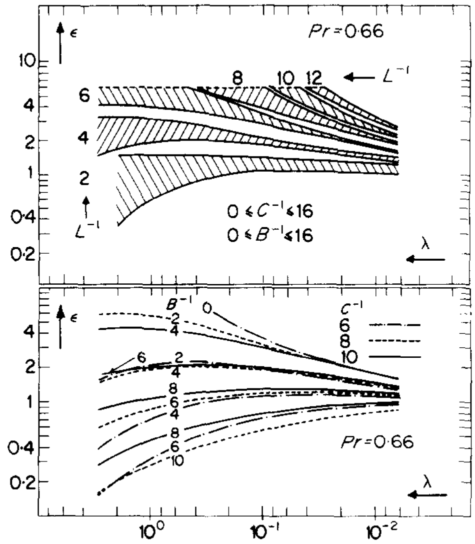

FIG. 4. Efficiency of a rough surface as a function of λ and roughness parameters.

L^{-1}. Furthermore, to compare different rough surfaces in different channels we need only compare values of L^{-1} at the same λ. This is a rigorous method of comparison. Comparing values of ε for different roughnesses at the same channel Reynolds number, the usual method, is conceptually not a valid comparison.

5. PREDICTIONS OF THE ROUGH-SURFACE PARAMETERS R, g, B^{-1}, C^{-1}, AND L^{-1}

For a range of representative geometry parameters h^+, p/h, h/b, for some representative values of c/h and C_D and for $Pr = 0{\cdot}66$, values of R and g were calculated [5]. B^{-1}, C^{-1} and L^{-1} were given from equations (9), (13) and (16). The results are presented in Figs. 5–10, where the chosen geometry parameters are specified. To avoid confusion some curves have been omitted.

Values of R and B^{-1} presented in Figs. 5 and 6 may be used to obtain St, λ and $Re(D)$, for a particular D/h, from Fig. 3. The value of R at $h^+ = 400$ may be considered representative within the range $100 < h^+ < 1000$. B^{-1} and values of R outside this range may be interpolated.

The analysis indicated that maximum values of ε occurred at $h^+ \approx 20$, over the complete range of geometry parameters. This has also been suggested by experiment [2]. In Figs. 7–9 B^{-1} and C^{-1} are given. These may be used in conjunction with Fig. 4 or with equations (2), (6) and (12) to determine ε accurately. Alternatively, values of L^{-1} presented in Fig. 10 may be employed, together with Fig. 4, as a guide to ε and an optimum roughness.

6. CONCLUDING REMARKS

Performing experiments to determine the thermohydraulic performance of a rough surface is a difficult and expensive procedure, further complicated by the wide range of different correlating procedures that may be employed. Here, by means of an analysis based upon

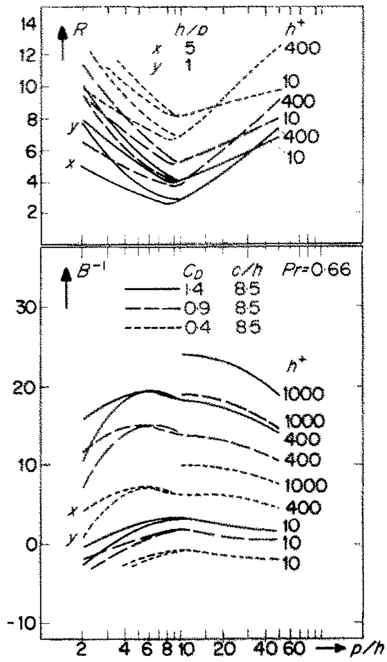

FIG. 5. Roughness parameters R and B^{-1} as a function of roughness shape and distribution.

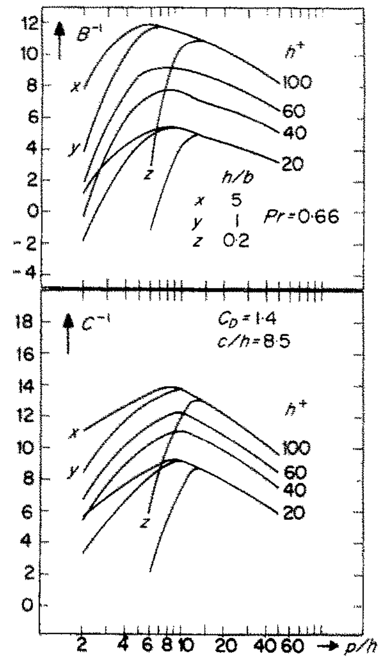

FIG. 7. Roughness parameters B^{-1} and C^{-1} as a function of roughness shape and distribution.

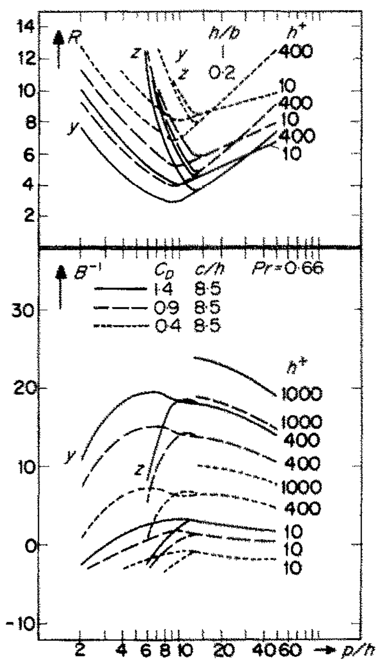

FIG. 6. Roughness parameters R and B^{-1} as a function of roughness shape and distribution.

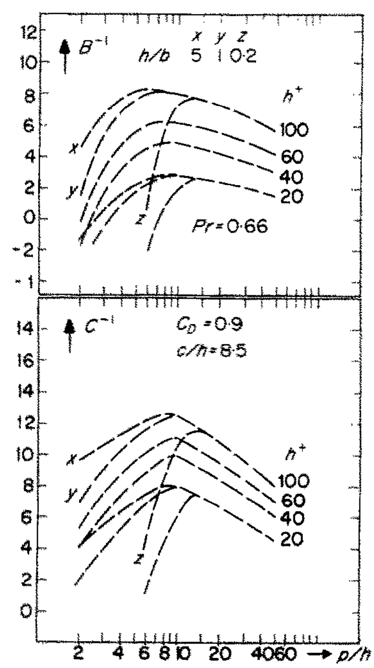

FIG. 8. Roughness parameters B^{-1} and C^{-1} as a function of roughness shape and distribution.

an approximate model of the separated flow over each roughness element, we have tried to unify the approach to rough surfaces. A new optimization parameter L^{-1} has been defined and design charts for a wide range of representative surfaces have been presented. These charts should be considered as a guide and as a basis for choosing future experiments; either to improve the theoretical model or to modify the present predictions. We may use the results presented here to extrapolate experimental information and to investigate off-design situations. Also, we can draw some tentative conclusions.

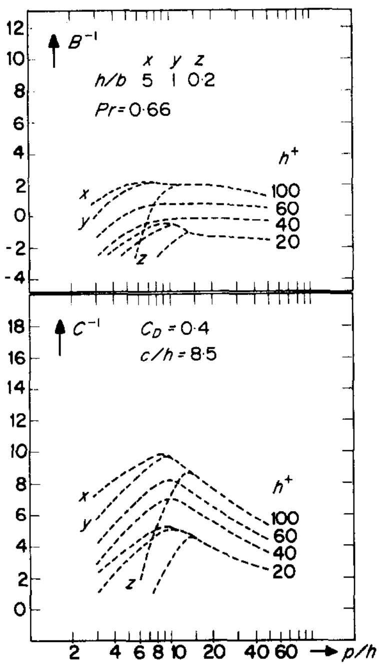

FIG. 9. Roughness parameters B^{-1} and C^{-1} as a function of roughness shape and distribution.

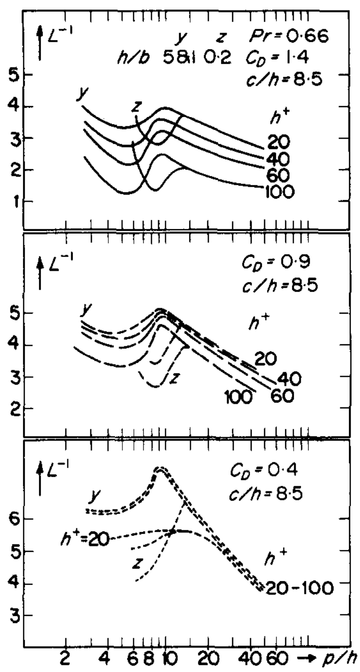

FIG. 10. The efficiency parameter L^{-1} as a function of roughness shape and distribution.

To achieve high Stanton numbers, irrespective of the friction factor, low values of R coupled with a small D/h are required. This is best achieved with a roughness having a high C_D—sharp edged ribs, say—with a cavity width $p-b \approx c$. Then operate at as low a value of h^+, to give a small B^{-1}, consistent with the requirements of a hydraulically rough surface.

For an optimum efficiency the situation changes. A value of $h^+ \approx 20$ appears to be an optimum condition, and sharp edged ribs are not the most efficient. It seems that L^{-1} and hence ε increases with decreasing C_D. Not too much emphasis is placed on this trend at C_D values much lower than about 0·9 because the separated flow regions for streamlined bodies are not similar to those found on rectangular ribs. A C_D around 0·9 is probably the most efficient with, again, $p-b \approx c$. As the roughness is more closely packed, or p/h is reduced, a tendency is evident for L^{-1} to increase above its value at $p-b \approx c$. Again not too much emphasis is placed on this trend because L^{-1} is formed from a difference between two numbers of similar magnitude, both of which are very sensitive to the geometry of the closely packed elements. These trends can only be established by careful experiments.

It appears that stagnation regions play the most important role as far as augmenting the heat transfer from a rough surface [5]. However, the number of stagnation regions which can be incorporated per unit area of surface is limited by the re-attachment of the separated flows; if elements are too closely spaced the preceding element shields the next element downstream. As a suggestion for a possible hybrid, efficient roughness form Fig. 11 is presented. For an efficient surface we need to turn the flow towards the surface to form stagnation regions without paying for the loss of momentum or form drag caused by entrainment. The entrainment could be replaced by an efficient aerofoil which turns the flow as shown.

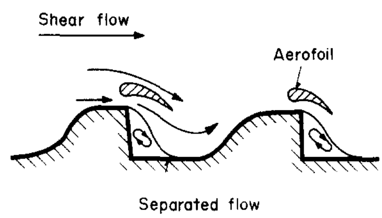

FIG. 11. A hybrid roughness form.

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OPTIMISATION DU REGIME THERMOHYDRAULIQUE DES SURFACES RUGUEUSES

Résumé—Les paramètres qui déterminent le régime des transferts de quantité de mouvement et de chaleur entre un fluide et une paroi rugueuse dans un canal à section constante sont présentés d'une façon originale. On définit un nouveau paramètre d'efficacité pour optimiser ce régime. En utilisant une nouvelle méthode d'analyse, récemment développée, on examine une large gamme de surfaces rugueuses et on présente les résultats sous forme graphique.

OPTIMIERUNG DER THERMOHYDRAULISCHEN AUSBILDUNG RAUHER OBERFLÄCHEN

Zusammenfassung—Die Parameter, welche das Strömungs- und Wärmeübergangsverhalten einer rauhen Oberfläche in einem gleichförmigen Kanal beschreiben, werden auf eine neue Art dargestellt. Für die Optimierung rauher Oberflächen wird eine neue Wirkungsgrad-Kenngröße definiert. Ein großer Bereich von rauhen Oberflächen wird mit Hilfe einer neu entwickelten analytischen Methode untersucht, und die für die Berechnung notwendigen Kenngrößen werden grafisch dargestellt.

ОПТИМИЗАЦИЯ ТЕРМОГИДРАВЛИЧЕСКИХ ХАРАКТЕРИСТИК ШЕРОХОВАТЫХ ПОВЕРХНОСТЕЙ

Аннотация — По-новому представлены параметры, описывающие характеристики переноса количества движения и тепла шероховатых поверхностей при однородных течениях в каналах. Определяется новый параметр эффективности при оптимизации этих характеристик. С помощью недавно разработанной методики исследуется целый ряд шероховатых поверхностей и приводятся расчетные графики.